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Application of the sequential analysis method in the justification of optimal managerial decisions in the context of uncertainty

Aplicación del método de análisis secuencial en la justificación de decisiones gerenciales óptimas en contexto de incertidumbre

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ABSTRACT:
The possibility of the practical application of the sequential analysis method (SAM) in making a decision on the feasibility of using new tactical techniques of actions for the fire emergency response units (FERU) has been explored. It has been demonstrated that this method allows to justify the choice of measures depending on the magnitude of a certain probability characteristic using the results of a limited number of tests. The formulas on which the method is based have been substantiated.
Keywords: mathematical modeling; sequential analysis; verification of statistical hypothesis, managerial decisions.

RESUMEN:
Se ha explorado la posibilidad de la aplicación práctica del método de análisis secuencial (SAM) para tomar una decisión sobre la viabilidad de usar nuevas técnicas tácticas de acción para las unidades de respuesta de emergencia contra incendios (FERU). Se ha demostrado que este método permite justificar la elección de medidas según la magnitud de una cierta característica de probabilidad utilizando los resultados de un número limitado de pruebas. En contraste con los métodos clásicos de estadística matemática, se requieren significativamente menos observaciones para sacar conclusiones, y su número no se determina de antemano. Las fórmulas en que se basa el método han sido fundamentadas.
Palabras clave: modelación matemática; análisis secuencial; verificación de hipótesis estadísticas, decisiones gerenciales

1. Introduction

Development and adoption of managerial decisions by the authorities (officials, commanders (heads)) of the executive department of the Russian Federation government are aimed at developing ways of using subordinate forces and measures that are optimal in the expected conditions of the situation.

Undoubtedly, the solution formation remains a deeply creative process. However, mathematical modeling, the efficiency of which has been repeatedly confirmed in various fields of applied science over the years, helps developing possible ways of using forces and measures and choosing the optimal one, (Akbar & Beg, 2016; Fitzgibbon *et al*., 2014; Siddiqi *et al*., 2015; Giordano *et al*., 2013; Habib, 2016; Meerschaert, 2013; Yang, 2013; Batkovskiy *et al.*, 2016; Gorbunov & Vasilieva, 2014; Gorbunov *et al.*, 2017; Malygin & Schetka, 2014).

The mathematical modeling methods allow a comparative assessment of the efficiency of various planned options for action (alternatives). The remarkable property of mathematical models allows to estimate the impact of various elements of the situation on the selected efficiency indicator, taking such interconnections and interdependencies between them into account, which the human brain cannot estimate. This allows to quantitatively estimate the best options for action (the best alternatives) from among those considered by a complete enumeration of all possible options. However, this is not always feasible.

This is primarily due to the limiting factors that describe situations where decisions should be formulated and made. These factors can be divided into three main groups (Motorygin & Galishev, 2013; Malygina & Marin, 2013):

- Economic factors (money, production and human resources, time, etc.);
- Technical factors (dimensions, weight, power consumption, reliability, accuracy, etc.); and
- Social factors that account for the human ethics and morality, ensuring the safety of people, and environmental requirements.

As such, the number of compared options can be so large that the enumeration cannot be completed within the available time.

In many cases, uncertainty may arise at the time of making a decision regarding the true state of the number of elements of the situation. In this case, identifying the best course of action by simple sequential comparison of all the options will be impossible.

These circumstances necessitate the use of mathematical optimization methods for the quantitative justification of decisions. Such methods, as well as the decision-making models obtained with their help, can be divided into the following groups:

- Methods and models used in conditions of certainty (reliable knowledge of situation elements) (Balychev *et al.*, 2018);
- Methods and decision-making models used in the context of risk. The following criteria are applied for choosing the "optimal solution" in these models: a) minimization of expected losses (Batkovskiy *et al.*, 2017); b) maximization of the expected result expressed as cash payments; and c) maximization of the expected result as a utility value (Motorygin & Galishev, 2013); and
- Methods and models used in the context of uncertainty regarding the true state of the situation elements.

In the context of uncertainty, the operational tactical formulation of the simulation problem should contain all the information required for the mathematical formulation of the problem and the choice of the optimization method.

First of all, it is required to formulate the purpose of using forces and measures. This needs to be done to determine the performance indicator. After that, the optimization purpose and the way to implement the plan are indicated. The optimization structure may vary, depending on whether the plan can be adjusted during its implementation.

After that, it is advisable to present the following factors:

- Need to choose one of several options for action; and
- Possibility of several situation options when the task is performed by forces.

In this case:

- It is unknown in what situation the task will have to be performed at the time of making a decision; and
- Various options of actions are optimal at various options of an expected situation.

Both the number of options for action and the number of options for the situation can actually be finite (but not less than two) and infinite.

Next, the factor creating uncertainty is indicated, as a rule. For example, it may be the lack of knowledge of the true state of any elements of the situation as a result of objective causes, independent of the human will and consciousness.

Making a decision on the adoption of a new complex technical system, a sample of special technique, a new sample technique (ST) of measures when performing a task depends on the knowledge of probabilistic characteristics in some cases. They can be the probability of a failure-free equipment operation, probability of completing a task, dispersion of a normally distributed random variable, etc. At the same time, the required probability characteristic is unknown and can only be estimated through a specially arranged experiment (training, test).

It is known (Wentzel, 2009; Wentzel, 2010; Kremer, 2012) that the researcher does not receive the true values of random variables, but their statistical estimates, when processing the experimental data. The more the data are processed, the more accurate and reliable these estimates are. However, better accuracy and reliability can be associated with the influence of such limiting factors as the large waste of effort, money, and time for testing. Besides, the increase in the specimens' testing, especially expensive and single-use, for determining suitability for use (arming), leads to the decrease in their number. Better accuracy and reliability of statistical estimates for such equipment samples is not always possible and appropriate to achieve through more tests.

In such cases, SAM developed by the Hungarian mathematician Wald (1947), is a fairly efficient mathematical method. It allows to justify the choice in conditions when it depends on the knowledge of some unknown probabilistic characteristic, where a limited number of tests can be conducted for its determination or estimation.

2. Methods

The key advantage of the SAM in comparison with the classical methods of mathematical statistics is that it requires significantly smaller number of observations, which allows to make a guaranteed scientifically grounded managerial decision, provided the above conditions and restrictions.

Unlike other statistical methods, the required number of observations is not determined in advance in the SAM, and the results of the experiment are evaluated after each test. In this case, two hypotheses are considered: about conformity ( ) and nonconformity ( ) of the process under study with the requirements (sample of a complex technical system, ST of forces, achieved level of training, etc.). These requirements are set by some probabilistic characteristic.

One of three solutions is recommended after each test:

- Accept an ST as complying with the requirements (implementation of hypothesis).
- Reject an ST (implementation of hypothesis).
- Conduct another test, because the obtained information is insufficient for accepting or rejecting hypothesis or .

If the first or the second decision is made, the experiment ends; if the third decision is made, then it continues. Therefore, the number of tests is a random variable.

The sequential analysis does not allow to completely remove the uncertainty regarding the true value of the required probability characteristic. In this regard, it can be recommended to accept an unsuitable sample of equipment or reject the best ST when performing the task, according to test data. The fewer are the observations, the greater is the possibility of such errors.

To determine the SAM scope, the specific features are required that distinguish it from the already known features. These features can be identified if the essence of the method is known. The following reasoning can be provided to clarify it.

The requirements for the technique sample or any process on reliability, probability of completing the task, dispersion, etc. should be determined prior to the experiment.

Let us review the possibility of mathematical modeling for making scientifically based decisions on the adoption of a new ST of actions based on checking its compliance with the performance requirements (Kamenetskaya *et al*., 2017a).

3. Results

3.1. Justification of a managerial decision on the efficiency of the new sample technique for fire fighting and rescue efforts

One of the key areas contributing to the solution of the EMERCOM FERU tasks is the development of new sample techniques (STs) for conducting operational actions on fires and in the aftermath of emergency situations (ES).

Undoubtedly, the adoption of new STs of action by FERU should be preceded by a series of experiments (training) proving their higher efficiency (according to specified criteria) compared to the existing ones.

However, the limiting factors that have significant impact on the possibility of experiments related to the use of such methods of FERU actions in various dangerous situations include significant waste of effort, time, and money. In addition, the experiment is impossible in cases where it is associated with a threat to the life or health of people.

Problem setting. Based on the analysis of the FERU operational activities, a new ST has been developed to fight fires and conduct rescue efforts (eliminating emergency consequences).

The requirements are determined that a new ST must meet (Rodionov, 2003). For example, the probability of fighting a fire in a standard time is specified or the mathematical expectation of the maximum prevented material damage caused by an emergency (fire), which must be not less than a specified value. To estimate the efficiency of a new ST, it should be tested in training (tests). The expediency of adopting a new ST should be determined on the basis of a limited number of such trainings with the SAM use.

The probability that fire will be extinguished in time not exceeding the given is taken as a probability characteristic and performance criterion:

$$P(t_{FF} \le t_{set}) = W \ge P_{set},$$

where P_{set} is the smallest permissible value of the probability of completing the task in the considered conditions of the operational situation in the implementation of the studied ST; t_{FF} is time to fight the fire when using the studied ST; t_{set} is the limiting time to fight the fire set by experts; *W* is the criterion of ST efficiency (Terebnev & Terebnev, 2003).

In other words, the ST is deemed expedient if it corresponds to the probability of completing the task $W \ge P_{set}$, and inexpedient if $W < P_{set}$.

It must be noted that the given time to fight the fire is not included in the task of this study; it can be specified on the basis of statistical data obtained from studies of fighting real fires, for example, (EMERCOM of Russia, 2010).

As has already been noted, the required number of observations is not established in advance. The results of each training are sequentially analyzed, and one of three decisions is recommended based on the analysis:

1) Consider the ST efficient according to a given criterion and adopt it (implementation of hypothesis H_0);
2) Reject the ST, consider it unsuitable for the task (implementation of hypothesis H_1); or
3) Conduct another test, as the information received is not sufficient to accept or reject hypothesis H_0 or H_1.

If the first or the second decision is made, then the experiment ends; if the third decision is made, then it continues. Therefore, the number of tests is a random variable.

To build formulas that allow statistical evaluation of the probabilistic characteristic under study and obtain its boundary values, let us consider the conditional process of sequential analysis when checking the ST for compliance with the requirements (Kamenetskaya *et al.*, 2017a).

Based on experience, conditions, and regulatory requirements for the ST, a certain threshold is established for the probability p' that the ST may be unsuitable. If it turns out that the true value p of the proportion of failed tests of the studied ST is less then p', then it is concluded that the ST meets the requirements, is efficient, and can be adopted (hypothesis H_0 is accepted). When $p > p'$, the ST should be considered unsuitable for the task of fighting fires.

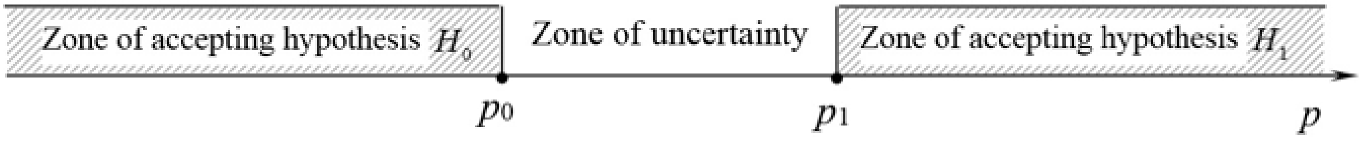
In the context of uncertainty about the exact knowledge of the probability that the ST will be inefficient, the possibility of making a mistake is allowed. It is possible that a decision may be made to reject an efficient ST or to adopt an ST that does not meet the requirements. The more significantly the established threshold value p' differs from the exact value p, the more significant are the mistakes in recommending whether to accept or reject the ST. If these values are close ($p \approx p'$), then the mistakes are noncritical.

As such, a certain zone of indifference to the indicated mistakes (Wald, 1947; Volgin *et al.*, 1981) is created around the threshold value p'. Let us establish the lower and upper limits of the zone, beyond which such mistakes are unacceptable, as probabilities p_0 and p_1, where $p_0 < p$, $p_1 > p'$.

In this case, three zones relative to the threshold probability value can be specified (Figure 1):

- Zone of accepting ST, $p' \le p_0$;
- Zone of rejecting ST, $p' \ge p_1$
- Zone of indifference (uncertainty), $p_0 < p' < p_1$.

Figure 1
Zones of values of the possible
threshold probability *p'*



It is assumed that a mistake of the first kind is made if the correct hypothesis H_0 is rejected, under the conditions of this task, the ST that meets the requirements is not accepted, with a small share of unsuccessful trainings ($p \le p_0$), as well as a mistake of the second kind, i.e. unsuitable hypothesis H_0 is adopted, as well as an unsuitable ST at $p \ge p_1$.

In each task of testing statistical hypotheses, the permissible probabilities α and β of making mistakes of the first and second kind are established, depending on the severity of their consequences. In each case, the numerical values of α and β, as well as p_0 and p_1, should be established using the experience of previous observations and the conditions of specific tests.

As such, the permissible risk is determined by values p_0, p_1, α and β. If they are given, then it is necessary to require in the ST testing that the probability of finding it unsuitable at $p \le p_0$ is no more than α, and the probability to recommend the ST for use at $p \ge p_1$ is no more than β. Then the boundary conditions can be formulated as follows: the probability of rejecting the ST at $p = p_0$ is equal to α, and the probability to consider adopting the ST expedient at $p = p_1$ is equal to β.

For the mistakes of the first type, the values of α = 0.005; 0.01; 0.05; 0.10 are standard, although any other can be selected as well. In technical studies, α = 0.05 is most often taken, and in studies closely related to the risk to human life and health, α = 0.01 is most often taken.

Within the framework of the above task, the ST is considered efficient for fighting fires if the percentage (share) of trainings with an unsuccessful result p does not exceed p_0, and inefficient if p is not less than p_1.

The purpose of modeling is to develop recommendations for making decisions about the efficiency or inefficiency of the ST during training, guided by the SAM.

To develop a mathematical model for testing a new ST, let us suppose that *n* trainings have been conducted, of which *m* turned out to be unsuccessful and (*n* – *m*) turned out to be successful (Volgin *et al.*, 1981; Kamenetskaya *et al.*, 2017a; 2017b).

At the share of unsuccessful trainings *p,* the probability of such a set of successful and unsuccessful trainings is

$$P_n(p) = p^m \cdot (1-p)^{n-m} \quad (1)$$

This probability is called the *likelihood function* of the hypothesis that the share of unsuccessful trainings is equal to *p.*

Let us consider the values of function (1) at the upper and lower boundaries of the uncertainty zone: $P_n(p_1)$ and $P_n(p_0)$ are probabilities that from *n* tests exactly *m* of them fail, if $p = p_1$ and $p = p_0$ (Figure 1).

The *likelihood ratio* μ is used as a criterion for sequential testing (Wald, 1947; Kamenetskaya *et al.*, 2017a):

$$\mu = \frac{P_n(p_1)}{P_n(p_0)}. \quad (2)$$

As such, the likelihood ratio is equal to the ratio of the probability of realization of hypothesis H_1 to the probability of the realization of hypothesis H_0.

The larger μ is, the more there is the reason to recognize the tested ST inefficient for successfully fighting fires. With a small μ, a new ST should be considered as complying with regulatory requirements and recommended for use in the operational activities of the fire department. The test stops in both cases. If ratio μ takes some intermediate value, then another training is required.

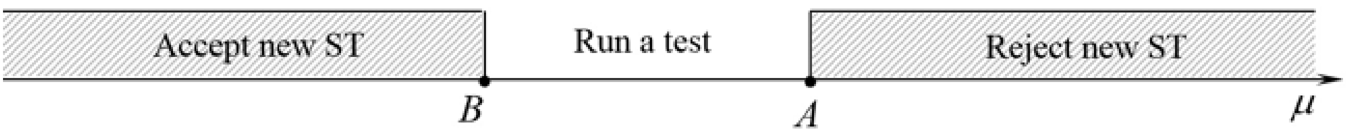
The principle of sequential analysis with the calculation of the likelihood ratio is as follows (Figure 2) (Wald, 1947; Volgin *et al.*, 1981; Kamenetskaya *et al.*, 2017a):

at $\mu \ge A$ new ST is rejected as not meeting the requirements (hypothesis H_1 is true);

at $\mu \le B$ new ST is accepted (hypothesis H_0 is true);

at $B < \mu < A$ a new test is required.

Figure 2
Zones of values of ratio μ when checking a new ST



Let us define the limiting values *A* and *B* as a function of the probabilities of making mistakes of the first and second kind.
The new ST is rejected due to the fact that it does not meet the requirements, and the second kind of mistake has not been made with probability $1-\beta$ at the upper boundary, or due to the first kind of mistake at the lower boundary, the probability of which is equal to α. In this case:

$$\mu = \frac{1-\beta}{\alpha} \geq A. \tag{3}$$

A new ST is accepted when the second kind of mistake is made on the upper boundary with probability β or when it meets the requirements, and no first kind of mistake is made with probability $1-\alpha$. Then

$$\mu = \frac{\beta}{1-\alpha} \leq B. \tag{4}$$

The check should continue if

$$\frac{\beta}{1-\alpha} < \mu < \frac{1-\beta}{\alpha}. \tag{5}$$

As such, it can be denoted as follows:

$$A = \frac{1-\beta}{\alpha};\ B = \frac{\beta}{1-\alpha}. \tag{6}$$

Let us determine the dependence of *m* number of unsuccessful trainings on the random number of tests *n* and on the values p_0, p_1, α and β.

Denote as follows: $q_1 = 1 - p_1;\ q_0 = 1 - p_0$.
Next, using formulas (1), (2) and (6), inequality (5) is transformed as follows:

$$\frac{\beta}{1-\alpha} < \left(\frac{p_1}{p_0}\right)^m \cdot \left(\frac{q_1}{q_0}\right)^{n-m} < \frac{1-\beta}{\alpha}. \tag{7}$$

Logarithm inequality (7) by performing the necessary transformations using the properties of logarithms (Wald, 1947; Volgin *et al.*, 1981; Kamenetskaya *et al.*, 2017a) to get a new inequality:

$$\frac{\ln\frac{\beta}{1-\alpha}}{\ln\left(\frac{p_1}{p_0}\cdot\frac{q_0}{q_1}\right)} + n\frac{\ln\left(\frac{q_0}{q_1}\right)}{\ln\left(\frac{p_1}{p_0}\cdot\frac{q_0}{q_1}\right)} < m < \frac{\ln\frac{1-\beta}{\alpha}}{\ln\left(\frac{p_1}{p_0}\cdot\frac{q_0}{q_1}\right)} + n\frac{\ln\left(\frac{q_0}{q_1}\right)}{\ln\left(\frac{p_1}{p_0}\cdot\frac{q_0}{q_1}\right)}. \tag{8}$$

Introduce the following:

$$a = \frac{\ln\frac{1-\beta}{\alpha}}{\ln\left(\frac{p_1}{p_0}\cdot\frac{q_0}{q_1}\right)}; \qquad b = \frac{\ln\frac{\beta}{1-\alpha}}{\ln\left(\frac{p_1}{p_0}\cdot\frac{q_0}{q_1}\right)}; \qquad k = \frac{\ln\left(\frac{q_0}{q_1}\right)}{\ln\left(\frac{p_1}{p_0}\cdot\frac{q_0}{q_1}\right)}. \tag{9}$$

Then inequality (8) can be written as follows:

$$b + nk < m < a + nk. \tag{10}$$

Inequality (10) allows to formulate recommendations for making a decision during the tests, depending on the number of unsuccessful trainings using the new ST:

1) Reject the new ST at $m \geq a + nk$;
2) Accept the new ST at $m \leq b + nk$;
3) Conduct another test at $b + nk < m < a + nk$.

3.2. Practical implementation of the SAM when testing the new ST. Graphic interpretation of the method

The application of this form of the method is carried out through building a special chart before the experiment (Wald, 1947; Kamenetskaya *et al.*, 2017a; 2017b). To this end, values *W*, p_0, p_1, α and β are first set.

For example, it is considered that the ST is expedient if it corresponds to the probability of completing the task $W \geq 0.9$ and is inexpedient at $W < 0.9$ (Diner, 1969).

Then, the following requirements are set to determine the zone of uncertainty and the allowable risk associated with the wrong decision:

Decision that the ST is advisable at $W < 0.8$ should be taken with the probability of no more than 0.30;

Decision that the ST is inappropriate at $W > 0.96$ should be taken with the probability of no more than 0.05.

Based on these conditions, p', p_0 and p_1 at $\alpha = 0.05$; $\beta = 0.3$ are found.

$p' = 1 - 0.9 = 0.1$;

$p_0 = 1 - 0.96 = 0.04$;

$p_1 = 1 - 0.8 = 0.2$.

Then, the values of *a*, *b* and *k* are determined using formulas (9) for the indicated values of p_0, p_1, α and β, and two parallel straight lines are built corresponding to equations $m_0 = b + kn$ and $m_1 = a + kn$. Since $p_0 \leq p_1$ is constant, then $k < 1$, i.e. the angle of inclination of the straight lines m_0 and m_1 to the abscissa axis is less than 45°.

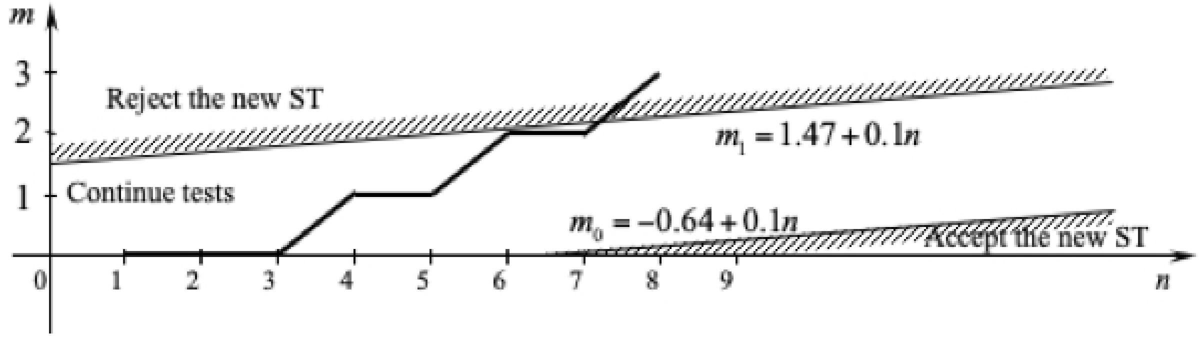
Let us demonstrate how to use the chart by example, with $p_0 = 0.04$; $p_1 = 0.2$; $\alpha = 0.05$; $\beta = 0.3$.

In accordance with formulas (9) $a = 1.47$; $b = -0.64$; $k = 0.10$.

Straight lines $m_1 = 1.47 + 0.1n$ and $m_0 = -0.64 + 0.1n$ (Figure 3) cut off segments $a = 1.47$ and $b = -0.64$ on the *m* axis and form angle 5.7° with the positive direction of the x-axis. The point of intersection of straight line $m_0 = -0.64 + 0.1n$ with the horizontal axis has coordinates (6.4; 0). A zone of uncertainty lies between parallel lines m_0 and m_1. As such, the point (*n*; *m*) hitting the area to the left and above line m_1 corresponds to the case when the new ST should be rejected, and hitting the area to the right and below line m_0 corresponds to the case when it should be accepted.

A sequential check then begins. Suppose that the first training with the use of a new ST be successful, which corresponds to a point with coordinates (1; 0), which lies in the zone of uncertainty. Therefore, the next training is required. Suppose that the ST also met the requirements in the second and third trainings. Points (2; 0), (3; 0) lie on the *n* axis, and, therefore, fall into the area of uncertainty, which means that the tests should continue. Suppose that the fourth training with the use of the new ST proved unsuccessful. A link with a vertex at the point (4; 1) is added to the line passing through points (1; 0), (2; 0), (3; 0). The process continues until the polyline connecting the points $(n_i; m_i)$ (where *i* is the number of training) does not cross line m_0 or m_1.

Figure 3
Chart of the sequential analysis of the new ST



The chart in Figure 3 illustrates an example of sequential analysis when the new ST failed to meet the performance requirements in the fourth, sixth and eighth trainings. In this case, after the eighth test, the polyline crosses straight line $m_1 = 1.47 + 0.1n$ and falls in the zone of making a decision to reject the new ST. As such, eight trainings were needed in our example to make a decision on the efficiency of the new ST of the forces' actions.

3.3. Tabular interpretation of the method

Let us demonstrate the possibility of using tabular SAM form for a sequential check on the suitability of the new ST (Table 1) Kamenetskaya *et al.*, 2017a, 2017b).

In this case, the values of m_1 and m_0 for a certain number of trainings *n* should be found prior to the start of the study: $m_1 = 1.47 + 0.1n$ and $m_0 = -0.64 + 0.1n$. m_0 is called the acceptance number, and m_1 is called the rejection number (Kamenetskaya, *et al.*, 2017a). When filling a table, it makes sense to include only positive numbers in a column with m_0 values, since the number *m*, which will be compared with m_0, is always nonnegative.

In the course of sequential analysis, the number of trainings carried out at the moment *m*, which do not meet the requirements, is found in each test. The results are recorded in the table.

As with the use of the graphical method, the verification continues until $m_0 < m < m_1$. The check is terminated in the first case of noncompliance with the specified inequality, namely:

If $m \le m_0$, it is recommended to adopt the new ST for use in the operational activities of fire departments;

If $m > m_1$, it is recommended to reject the new ST.

Table 1

Data of possible test option

Test No. *n*	Acceptance number	Number of trainings with a failure *m*	Rejection number
1.	–	0	1.57
1.	–	0	1.67
1.	–	0	1.77
1.	–	1	1.87
1.	–	1	1.97
1.	–	2	2.07
1.	0.06	2	2.17
1.	0.16	3	2.27

As a result of a sequential analysis after the eighth check, a decision was made that the new ST of the actions of the FERU did not meet the requirements for the fire fighting efficiency criterion; therefore, more efficient ST should be considered.

3.4. Operational characteristics of the SAM criterion

An important characteristic of the described process is the probability that it will end with the adoption of one of two hypotheses, i.e., with the conclusion that the ST should be considered efficient and recommended for use, for example.

This probability is called the power function of a test in the literature devoted to the SAM (Wald, 1947).

As such, the probability of adopting new ST as a result of a sequential analysis, provided that the true value of the failed tests' share for the studied ST is equal to p, is called the *power function of a test* and is indicated as $L(p)$. The likelihood that the adoption of the ST would be considered inappropriate in these conditions, is equal to $1-L(p)$.

If $p=0$, then the polyline in Figure 3 will coincide with the abscissa axis and necessarily cross the boundary $m_0=-0.64+0.1n$, i.e., the power function will take on value $L(0)=1$. However, if $p=1$, then the polyline coincides with the bisector of the first coordinate quarter and necessarily crosses the boundary $m_1=1.47+0.1n$, i.e., the power function will take on value $L(1)=0$.

Since the probability of the ST adoption is equal to β at $p=p_1$ and is equal to $1-\alpha$ at $p=p_0$, then $L(p_1)=\beta$, and $L(p_0)=1-\alpha$. Besides, it can be shown that for p equal to the angular coefficient *k* of straight lines $m_0=b+kn$ and $m_1=a+kn$, the probability of making a decision on the expediency of the ST is equal to $\frac{a}{a+|b|}$, i.e.,

$$L(k)=\frac{a}{a+|b|}.$$

As such, there are five special points for the power function:

$$L(p)=\begin{cases} 1 & at\ p=0 \\ 1-\alpha & at\ p=p_0 \\ \dfrac{a}{a+|b|} & at\ p=k \\ \beta & at\ p=p_1 \\ 0 & at\ p=1 \end{cases}.$$

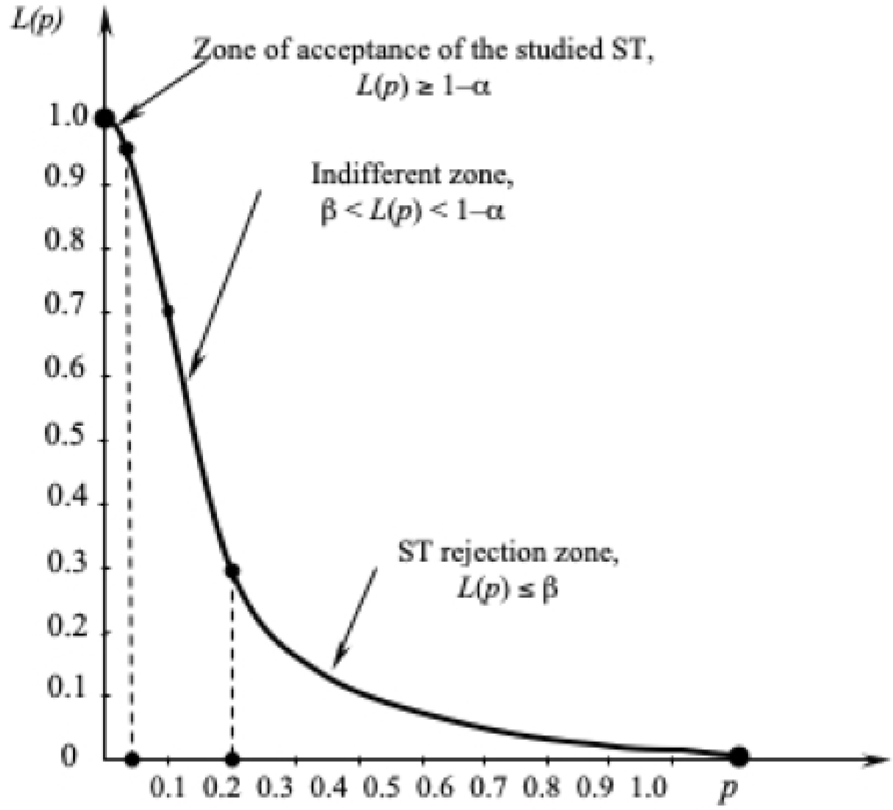
Using these points, a chart of function $L(p)$ can be built with sufficient accuracy.

In terms of our example,

$$L(p)=\begin{cases} 1 & at\ p=0 \\ 0.95 & at\ p=0.04 \\ 0.70 & at\ p=0.10 \\ 0.3 & at\ p=0.2 \\ 0 & at\ p=1 \end{cases}.$$

This power function is graphically shown in Figure 4.

Figure 4
A chart of the operational characteristics of the criterion in the sequential analysis of the feasibility of the new ST



The perfect and real operational characteristics can match only if the entire batch of products is checked or an infinite number of tests are carried out. However, this approach eliminates the use of the SAM.

As such, other things being equal, the method of testing the implementation of hypotheses H_0 or H_1 is more preferable in which the operational characteristic is closer to the perfect one with smaller number of tests. However, these arguments do not reduce the practical significance of the SAM.

Since the number of tests before making a certain decision is a random variable in a sequential analysis, the question arises of determining the mathematical expectation of the number of tests $M_p[n]$, which depends on the probability *p* of an unsuccessful outcome of the ST test. Under the conditions of this task, the average expected number of tests $M_p[n]$ is related to the power function of the test by the following approximate formula (Wald, 1947):

$$M_p[n] \approx \frac{L(p)\ln\dfrac{\beta}{1-\alpha}+\left(1-L(p)\right)\ln\dfrac{1-\beta}{\alpha}}{p\ln\dfrac{p_1}{p_0}+(1-p)\ln\dfrac{1-p_1}{1-p_0}}.$$

Since the *p* value is unknown, it can be assumed that $p = k \approx p'.$ for a preliminary estimate of the average expected number of tests.

In this case, approximately

$$M_k[n] \approx \frac{\ln\dfrac{1-\alpha}{\beta}\ln\dfrac{1-\beta}{\alpha}}{\ln\dfrac{p_1}{p_0}\ln\dfrac{1-p_0}{1-p_1}}.$$

In this example, $M_k[n] \approx 10.4$.

4. Discussion

The application of SAM in an applied task to develop an optimal managerial decision in the context of uncertainty has been demonstrated in the article, which is devoted to making a conclusion about the efficiency of new ST for fighting fires and rescue efforts.

The considered example of the practical application of the SAM illustrates the solution of this problem with the following results:

– Eight trainings were required with the use of the new ST of FERU actions to make a conclusion that this technique did not meet the requirements set by the criterion of efficiency in fighting fires, and more efficient ST should be considered;

– At selected values of α and β, the power of the criterion, determined by its power function, is quite high (Figure 4), and the approximate value of the average expected number of tests is 10.

The advantage of the SAM over the classical methods of mathematical statistics is shown in the particular example, which consists in the fact that this method allows to make a guaranteed scientifically-based managerial decision with a small number of tests in the context of uncertainty.

The formulas on which the SAM is based are substantiated. The possibilities of its practical implementation in two forms – graphical and tabular – are reviewed.

The SAM can be applied to solve a wide range of planning and management tasks in various areas:

– To develop optimal managerial decisions in the operational activities of the EMERCOM of Russia with little effort and resources;

– To conduct a comparative assessment of the efficiency of two actions or processes (Kamenetskaya *et al.,* 2017b);

– To check the batch of products for compliance with specifications;

– To develop recommendations for the acceptance of one of the competing types of equipment to be tested;

– To develop recommendations when checking of a normally distributed random variable with unknown dispersion or with unknown expectation for compliance with the requirements;

– To develop recommendations on the feasibility of adopting a new model of fire fighting equipment that has been modernized; and

– To check the compliance of the range of fire fighting equipment with the specifications.

The advantages of the SAM also include the relative simplicity of its practical application: the mathematical modeling of the above tasks requires only a statement of the success or failure of a particular test performed, which corresponds to two possible values of some random variable: 0 or 1.

5. Conclusion

The advantage of the SAM over other methods lies in its ability to significantly reduce the number of experiments required to collect statistical information. As such, in particular, there is the possibility of forming a guaranteed and scientifically grounded decision on the expediency of applying new STs of FERU actions with relatively little forces and measures.

There is a wide range of technical, economic, and military planning and management tasks similar to those reviewed, for which the use of the SAM ensures relative simplicity, accessibility, fairly high accuracy and reliability of conclusions (Diner, 1969; Kamenetskaya *et al*., 2017a; 2017b; Volgin *et al*., 1981).

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